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Multi-level Multi-objective Quadratic Fractional Programming Problem with Fuzzy Parameters: A FGP Approach

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Authors' contributions

This work was carried out in collaboration between all authors. Author MSO designed the study, performed the statistical analysis, wrote the protocol. Authors OEE and MAES wrote the first draft of the manuscript. Authors OEE and MAES managed the analyses of the study. Author MAES managed the literature searches. All authors read and approved the final manuscript.

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Abstract

The motivation behind this paper is to present multi-level multi-objective quadratic fractional programming (ML-MOQFP) problem with fuzzy parameters in the constraints. ML-MOQFP problem is an important class of non-linear fractional programming problem. These type of problems arise in many fields such as production planning, financial and corporative planning, health care and hospital planning. Firstly, the concept of the α-cut and fuzzy partial order relation are applied to transform the set of fuzzy constraints into a common crisp set. Then, the quadratic fractional objective functions in each level are transformed into non-linear objective functions based on a proposed transformation. Secondly, in the proposed model, separate non-linear membership functions for each objective function of the ML-MOQFP problem are defined. Then, the fuzzy goal programming (FGP) approach is utilized to obtain a compromise solution for the ML-MOQFP problem by minimizing the sum of the negative deviational variables. Finally, an illustrative numerical example is given to demonstrate the applicability and performance of the proposed approach.

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1 Introduction

Hierarchical optimization or multi-level mathematical programming (MLMP) techniques are extensions of Stackelberg games for solving decentralized planning problems with multiple decision makers (DMs) in a hierarchical organization where each unit seeks its own interests. The basic concept of multi-level programming technique is that the first-level decision maker (FLDM) sets his goal and/or make decision, set goal/or makes decision that requires each subordinate level in the organization for an independent optimal solution. These solutions are modified by the FLDM in line with the organizational objectives. This process proceeds to a satisfactory solution [1-5].

Over the last few years, rapid improvement in solving MLMP [2,3,6,7] as well as bi-level mathematical programming (BLMP) problems [1,8,9,10,5] have been witnessed and several methods have been presented. The use of the concept of the membership function of fuzzy set theory to multi-level programming problems for obtaining satisfactory decisions was first presented in [11]. FGP approach has been introduced in [12] for proper distribution of decision powers to the decision maker to arrive at a satisfying decision for the overall benefits of the organization. Sakawa et al. [5] proposed interactive fuzzy programming for two-level linear fractional programming problems with fuzzy parameters. FGP algorithm for solving a decentralized bi-level multi-objective programming problem was developed in [4]. Arora and Gupta [1] presented interactive FGP approach for linear bi-level programming problem with the characteristics of dynamic programming. Multi-level decision-making problems were recently studied in [3]. Pramanik and Roy [4] adopted fuzzy goals to specify the decision variables of higher level DMs and proposed weighted/ unweighted FGP models for solving MLMP to obtain a satisfactory solution. Also, FGP approach was extended for solving bi-level multi-objective programming problems with fuzzy demands [10].

The fractional optimization problem is one of the most difficult problems in the field of optimization. Optimization of the ratio of two functions is called fractional programming (ratio optimization) problem [13]. Indeed, in such situations, it is often a question of optimizing a ratio of output/employee, profit/cost, inventory/sales, student/cost, doctor/patient, and so on subject to some constraints [14,15]. A proposal to the solution of multi-objective linear fractional programming has been presented in [16]. Multi-objective quadratic fractional programming models involve optimization of many complex and conflicting objective functions in the mathematical form of quadratic fractional subject to the set of constraints. FGP approach for multi-objective quadratic fractional programming (MOQFP) problem has been presented in [17]. Such type of problems in large hierarchical organizations of complex and conflicting multi-objectives formulate ML-MOQFP problems. Recently Lachhwani [18] proposed FGP approach with some modifications for solving MOQFP model. An interactive FGP algorithm to solve decentralized bi-level multi-objective fractional programming problem was presented in [19]. Baky et al. [20] presented fuzzy goal programming procedures to bi-level multi-objective fractional programming. FGP approach to solve stochastic fuzzy multi-level multi-objective fractional programming problem was extended in [4]. Parametric multi-level multi-objective fractional programming problems with fuzziness in the constraints has been presented in [21].

During the past two decades, the majority of research on the multi-level programming problems have been concentrated on the deterministic version in which the coefficients and decision variables in the objective functions and the constraints are assumed to be crisp values. However, in reality, it is usually difficult to know precisely the values of the coefficients due to the existence of imprecise or uncertain information when establishing multi-level models [4]. Thus, lead us to present the current research hoping that the proposed ML-MOQFP problem with fuzzy parameters can contribute to future studies in the field of uncertain multi-level optimization.

This paper presents a FGP approach for solving ML-MOQFP problem with fuzzy parameters. These parameters are expressed as fuzzy numbers based on the fuzzy set theory [22] to account for the uncertainty in decision-making problems. This study also employs α-cut and fuzzy partial order relation to formulate the crisp model at the desired α-level.Then, the quadratic fractional objective functions in each level are transformed into non-linear objective functions based on a proposed transformation, thus the ML-MOQFP problem transformed into multi-level multi-objective non-linear fractional programming (ML-MONFP) problem. Secondly, separate non-linear membership functions for each objective function of the ML-MONFP problem are defined. Then, the FGP approach is utilized to obtain a compromise solution for the ML-MOQFP problem by minimizing the sum of the negative deviational variables. An algorithm for the ML-MOQFP problem is presented in details.

The remainder of this paper is organized as follows. Section 2 introduces some basic definitions and preliminary results. In Section 3, problem formulation of the ML-MOQFP problem with fuzzy parameters is exhibited and its equivalent deterministic model is formulated. Section 4 develops the non-linear model of the ML-MOQFP problem. The FGP approach for solving ML-MOQFP Problem with fuzzy parameters is introduced in Section 5. An algorithm for ML-MOQFP Problems with fuzzy parameters via FGP is proposed in Section 6. A numerical example to clarify the developed FGP approach is provided in Section 7.Concluding remarks are given at the end.

2 Preliminaries

In this section, some basic concepts and preliminary results used in this paper are briefly introduced.

2.1 Fuzzy number

Definition 1: Let R be the set of all real numbers. Then a real fuzzy number $\tilde{a}$ is defined by its membership function $\mu_{\tilde{a}}(x)$ that satisfies:

(1) A continuous mapping from R to the closed interval $[0, 1]$.
(2) $\mu_{\tilde{a}}(x) = 0$ for all $x \in (-\infty, a]$.
(3) Strictly increasing and continuous on $[a, b]$.
(4) $\mu_{\tilde{a}}(x) = 1$ for all $x \in [b, c]$.
(5) Strictly decreasing and continuous on $[c, d]$.
(6) $\mu_{\tilde{a}}(x) = 0$ for all $x \in [d, +\infty)$ [23].

Definition 2: a fuzzy number $\tilde{a}$ is said to be an $\mathcal{LR}$-fuzzy number if

$$\mu_{\tilde{a}}(x) = \begin{cases} \mathcal{L}\left(\dfrac{a-x}{\gamma^{a}}\right) & x < a, \quad \gamma^{a} > 0, \\ \mathcal{R}\left(\dfrac{x-a}{\beta^{a}}\right) & x > a, \quad \beta^{a} > 0, \end{cases} \tag{1}$$

where a is the mean value of $\tilde{a}$ and γ^{a} and β^{a} are positive numbers expressing the left and right spreads of $\tilde{a}$ and reference functions $\mathcal{L}, \mathcal{R}: [0,1] \to [0,1]$ with $\mathcal{L}(1) = \mathcal{R}(1) = 0$ and $\mathcal{L}(0) = \mathcal{R}(0) = 1$ are non-increasing, continuous functions [20].

Using its mean value and left and right spreads, and shape functions, such an $\mathcal{LR}$-fuzzy number is symbolically written as $\tilde{a} = (a, \gamma^{a}, \beta^{a})_{LR}$

Definition 3: The α-level set of the fuzzy parameter $\tilde{a}$, is defined as an ordinary set $L_{\alpha}(\tilde{a})$ for which the degree of its membership function exceeds the level set $\alpha \in [0,1]$, where [24,23]:

$L_{\alpha}(\tilde{a}) = \{a \in R^{m} | \mu_{\tilde{\mathbf{a}}}(x) \geq \alpha\} = \{a \in [\tilde{a}_{\alpha}^{L}, a_{\alpha}^{U}] | \mu_{\tilde{\mathbf{a}}}(x) \geq \alpha,\},$

where $\tilde{a}_{\alpha}^{L} = a - \gamma^{a}\mathcal{L}^{-1}(\alpha)$ and $\tilde{a}_{\alpha}^{U} = a + \beta^{a}\mathcal{R}^{-1}(\alpha)$.

For two $\mathcal{LR}$-fuzzy numbers $\tilde{a} = (a, \gamma^{a}, \beta^{a})_{LR}$ and $\tilde{b} = (b, \gamma^{b}, \beta^{b})_{LR}$ the formula for the extended addition becomes [24,4]:

(1) $(a, \gamma^{a}, \beta^{a})_{LR} + (b, \gamma^{b}, \beta^{b})_{LR} = (a + b, \gamma^{a} + \gamma^{b}, \beta^{a} + \beta^{b})_{LR}$,
(2) $(a, \gamma^{a}, \beta^{a})_{LR} - (b, \gamma^{b}, \beta^{b})_{LR} = (a - b, \gamma^{a} + \beta^{b}, \beta^{a} + \gamma^{b})_{LR}$
(3) $(a, \gamma^{a}, \beta^{a})_{LR} \times (b, \gamma^{b}, \beta^{b})_{LR} \cong (ab, a\gamma^{b} + b\gamma^{a}, a\beta^{b} + b\beta^{a})_{LR} \quad if \;\; a > 0,\; b > 0$,
(4) $\lambda(a, \gamma^{a}, \beta^{a})_{LR} = \begin{cases} (\lambda a, \lambda\gamma^{a}, \lambda\beta^{a})_{LR} & if \;\; \lambda \geq 0, \\ (\lambda a, -\lambda\beta^{a}, -\lambda\gamma^{a})_{RL} & if \;\; \lambda < 0, \end{cases}$ $\lambda$is a scalar

Throughout this paper, we shall take the ordering between two fuzzy numbers,$\tilde{a}$and$\tilde{b}$. According to the following definition.

Definition 4: Let $\tilde{a}_{\alpha} = [\tilde{a}_{\alpha}^{L}, \tilde{a}_{\alpha}^{U}]$ and $\tilde{b}_{\alpha} = [\tilde{b}_{\alpha}^{L}, \tilde{b}_{\alpha}^{U}]$ be two intervals. The order relations $\preccurlyeq_{LR}$ and $\prec_{LR}$ between $\tilde{a}_{\alpha}$ and $\tilde{b}_{\alpha}$ are defined as [25,26]:

1. $\tilde{a}_{\alpha} \preccurlyeq_{LR} \tilde{b}_{\alpha}$ if and only if $\tilde{a}_{\alpha}^{L} \leq \tilde{b}_{\alpha}^{L}$ and $\tilde{a}_{\alpha}^{U} \leq \tilde{b}_{\alpha}^{U}$,
2. $\tilde{a}_{\alpha} \prec_{LR} \tilde{b}_{\alpha}$ if and only if $\tilde{a}_{\alpha} \preccurlyeq_{LR} \tilde{b}_{\alpha}$ and $\tilde{a}_{\alpha} \neq \tilde{b}_{\alpha}$,

2.2 Compromise programming

Compromise programming, was introduced by Zeleny [27-29], seeks the compromise solution among the various objectives of a multi-criteria decision making problem. The idea is based on the minimization of the distance between the ideal and the desired solutions. To introduce the nomenclature, only the essential ideas are summarized [27,28,30]. Consider the multi-objective programming problem:

$$max \;\; \boldsymbol{F}(x) = [f_1(x), f_2(x), \dots, f_k(x)]^T \tag{2}$$

$$subject\ to \qquad x \in \boldsymbol{S} = \left\{ x \in R^n \middle| \begin{array}{ll} g_j(x) \leq 0, & j = 1,2, \dots, m, \\ h_l(x) = 0, & l = 1,2, \dots, e, \end{array} \right\} \tag{3}$$

where k is the number of objective functions, m is the number of inequality constraints, and e is the number of equality constraints. Also, $x \in R^n$ is a vector of decision variables, and $\boldsymbol{F}(x) \in R^k$ is a vector of objective functions $f_i(x): R^n \rightarrow R$.

The ideal solution of the above problem can be obtained by solving each objective function independently subject to the set of constraints. The vector of ideal solutions can be represented by $I^* = (f_1^*, f_2^*, \dots, f_k^*)$. Then the L_p- problem for minimizing the distance is now of the form:

$$min \left(\sum_{i=1}^{k} w_i^p |f_i^* - f_i(x)|^p \right)^{1/p} \tag{4}$$

$$subject\ to \qquad x \in \boldsymbol{S} \tag{5}$$

where p is the distance parameter with values $1 \leq p \leq \infty$, and w_i represent the weights of the objective functions or the normalizing factor. From a practical standpoint, the most important values for p are 1, 2 and ∞, for more details the reader is referred to [31,32,30].

Goal programming was first introduced by Charnes and Cooper [33], for more details the reader is refereed to [31,32,30]. Consider the general weighted goal programming (WGP) with minimization of d_i^-:

$$min \sum_{i=1}^{k} w_i \, d_i^- \tag{6}$$

$$subject\ to \qquad f_i(x) + d_i^- - d_i^+ = t_i^* \quad i = 1,2, \dots, k, \tag{7}$$

$$d_i^- \geq 0, \ \ d_i^+ \geq 0, \quad d_i^- \times d_i^+ = 0, \ \ i = 1,2, \dots, k, \tag{8}$$

$$x \in \boldsymbol{S} \tag{9}$$

Assume now that all targets are set to the ideal objective vector $t_i^* = f_i^*$, in the above model. As$d_i^+ = 0$, so $d_i^- = f_i^* - f_i(x)$, then by substituting this expression in the objective function (6). The WGP model (6)-(9) then turned into the compromise programming model (4)-(5) with $p = 1$. The L_1-metric is widely used in connection with goal programming because of the origin of the method in linear programming [31]. In the current research the FGP approach is extended to obtain a compromise solution for the ML-MOQFP problem with fuzzy parameters.

3 Problem Formulation

Multi-level programming problems have more than one decision maker. Consider the hierarchical system be composed of ap-level decision maker. Let the decision maker at the i^{th}-level denoted by DM_i controls over the decision variable $\boldsymbol{x_i} = (x_{i1}, x_{i2}, \dots, x_{in_i}) \in R^{n_i}, i = 1,2, \dots, p.$ where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_p}) \in R^n$ and $n = \sum_{i=1}^{p} n_i$and furthermore assumed that

$$F_i(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_p}) \equiv F_i(\boldsymbol{x}) \colon R^{n_1} \times R^{n_2} \times \dots \times R^{n_p} \to R^{k_i}, \quad i = 1,2, \dots, p, \tag{10}$$

are the vector of objective functions forDM_i, $i = 1,2, \dots, p$. Mathematically, ML-MOQFP problem with fuzzy parameters in the constraints [9,3,4,5] follows as:

$[\boldsymbol{1^{st}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_1}} F_1(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_1}} \left(f_{11}(\boldsymbol{x}), f_{12}(\boldsymbol{x}), \dots, f_{1k_1}(\boldsymbol{x}) \right), \tag{11}$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_p}\ \ solves$

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_2}} F_2(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_2}} \left(f_{21}(\boldsymbol{x}), f_{22}(\boldsymbol{x}), \dots, f_{2k_2}(\boldsymbol{x}) \right), \tag{12}$$

⋮

$where\ \boldsymbol{x_p}\ \ solves$

$[\boldsymbol{p^{th}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_p}} F_p(\boldsymbol{x}) = \underbrace{max}_{\boldsymbol{x_p}} \left(f_{p1}(\boldsymbol{x}), f_{p2}(\boldsymbol{x}), \dots, f_{pk_p}(\boldsymbol{x}) \right), \tag{13}$$

$subject\ to$

$$\boldsymbol{x} \in \tilde{G} = \{ \boldsymbol{x} \in R^n \,|\, \tilde{A}_1 \boldsymbol{x_1} + \tilde{A}_2 \boldsymbol{x_2} + \cdots + \tilde{A}_p \boldsymbol{x_p} \leq \tilde{\boldsymbol{b}}, \boldsymbol{x} \geq 0, \tilde{\boldsymbol{b}} \in R^m \}, \tag{14}$$

where objective functions $f_{ij}(\boldsymbol{x})$ are represented by a quaratic fractional function

$$f_{ij}(\boldsymbol{x}) = \frac{N_{ij}(\boldsymbol{x})}{D_{ij}(\boldsymbol{x})} = \frac{\boldsymbol{x}^T Q^{ij}\boldsymbol{x} + \boldsymbol{c}^{ij}\boldsymbol{x} + \alpha^{ij}}{\boldsymbol{x}^T R^{ij}\boldsymbol{x} + \boldsymbol{d}^{ij}\boldsymbol{x} + \beta^{ij}}, \quad i = 1,2,\dots,p, \quad j = 1,2,\dots,k_i. \tag{15}$$

where Q^{ij} is an $n \times n$ negative definite matrix,R^{ij} is an$n \times n$ positive semi-definite matrix$\boldsymbol{c}^{ij}, \boldsymbol{d}^{ij}$are n-vectors, $\tilde{\boldsymbol{A}}_{\boldsymbol{i}}$ is an$m \times n_i$, $i = 1,2,\dots,p$ fuzzy matrices and $\tilde{\boldsymbol{b}}$is an m-vector of fuzzy parameters.It is customary to assume that $D_{ij}(\boldsymbol{x}) > 0 \; \forall \; \boldsymbol{x} \in \tilde{G}$, also $\alpha^{ij}$and β^{ij} are constantsand $\tilde{G}$ represents the multi-level convex constraints feasible choice set in the fuzzy environment.

3.1 Formulation of crisp set of constraints and solution concept

Based on ML-MOQFP model $(11) - (14)$, the coefficients of the set of constraints are represented by fuzzy numbers. Let $\mu_{\tilde{\boldsymbol{A}}_i}$, and$\mu_{\tilde{\boldsymbol{b}}}$be the membership functions which representsthe fuzzy coefficients matrices$\tilde{\boldsymbol{A}}_{\boldsymbol{i}}$and the fuzzy numbers in the corresponding vector$\tilde{\boldsymbol{b}}$respectively. The α-cuts of$\tilde{\boldsymbol{A}}_{\boldsymbol{i}}and\tilde{\boldsymbol{b}}$ are defined as [10,3,34,35]:

$$(\boldsymbol{A}_{\boldsymbol{i}})_\alpha = \{\boldsymbol{A}_{\boldsymbol{i}} \in [(\boldsymbol{A}_{\boldsymbol{i}})^L_\alpha, (\boldsymbol{A}_{\boldsymbol{i}})^U_\alpha] | \mu_{\tilde{\boldsymbol{A}}_i} \geq \alpha, \boldsymbol{A}_{\boldsymbol{i}} \in S(\tilde{\boldsymbol{A}}_{\boldsymbol{i}})\}, \tag{16}$$

$$(\boldsymbol{b})_\alpha = \{\boldsymbol{b} \in [(\boldsymbol{b})^L_\alpha, (\boldsymbol{b})^U_\alpha] | \mu_{\tilde{\boldsymbol{b}}} \geq \alpha, \boldsymbol{b} \in S(\tilde{\boldsymbol{b}})\}, \tag{17}$$

where$S(\tilde{\boldsymbol{b}})$, and $S(\tilde{\boldsymbol{A}}_{\boldsymbol{i}})$ are the supports of the corresponding vectors and matrix of fuzzy numbers.

Let $\alpha \in [0,1]$, be the grade ofsatisfaction associated with the set of constraintsof the ML-MOQFP problem.the fuzzy constraintsin equation (14) are to be understood with respect to the ranking relation,$\sum_{j=1}^{n} \tilde{A}_{ij}\, x_j \leq_\alpha \tilde{b}_i$[25,26], between the fuzzy vectors which aregiven in the definition 4. Thus, for$\alpha \in [0,1]$, the feasible crisp set of theML-MOQFP problem can be described as:

$$\boldsymbol{x} \in \boldsymbol{G}_\alpha = \left\{ \boldsymbol{x} \in R^n \left| \begin{array}{ll} \sum_{j=1}^{n} (\tilde{A}_{ij})^U_\alpha x_j \leq (\tilde{b}_i)^U_\alpha, & x_j \geq 0 \\ \sum_{j=1}^{n} (\tilde{A}_{ij})^L_\alpha x_j \leq (\tilde{b}_i)^L_\alpha, & i = 1,2,\dots,m \end{array} \right. \right\} \tag{18}$$

Definition 5. For any $\boldsymbol{x}_{\boldsymbol{1}}(\boldsymbol{x}_{\boldsymbol{1}} \in (G_1)_\alpha = \{\boldsymbol{x}_{\boldsymbol{1}} | \boldsymbol{x} = (\boldsymbol{x}_{\boldsymbol{1}}, \boldsymbol{x}_{\boldsymbol{2}}, \dots, \boldsymbol{x}_{\boldsymbol{p}}) \in (G)_\alpha\})$ given by FLDM and $\boldsymbol{x}_{\boldsymbol{2}}(\boldsymbol{x}_{\boldsymbol{2}} \in (G_2)_\alpha = \{\boldsymbol{x}_{\boldsymbol{2}} | \boldsymbol{x} = (\boldsymbol{x}_{\boldsymbol{1}}, \boldsymbol{x}_{\boldsymbol{2}}, \dots, \boldsymbol{x}_{\boldsymbol{p}}) \in (G)_\alpha\})$ given by SLDM, if the decision variable $\boldsymbol{x}_{\boldsymbol{p}}\left(\boldsymbol{x}_{\boldsymbol{p}} \in (G_p)_\alpha = \{\boldsymbol{x}_{\boldsymbol{p}} | \boldsymbol{x} = (\boldsymbol{x}_{\boldsymbol{1}}, \boldsymbol{x}_{\boldsymbol{2}}, \dots, \boldsymbol{x}_{\boldsymbol{p}}) \in (G)_\alpha\}\right)$ is the α-Pareto optimal solution of the PLDM, then $(\boldsymbol{x}_{\boldsymbol{1}}, \boldsymbol{x}_{\boldsymbol{2}}, \dots, \boldsymbol{x}_{\boldsymbol{p}})$ is an α-feasible solution of ML-MOQFP problem.

Definition 6. If$\boldsymbol{x}^* = (\boldsymbol{x}_1^*, \boldsymbol{x}_2^*, \dots, \boldsymbol{x}_p^*)$ is an α-feasible solution of the ML-MOQFP problem; no other α-feasible solution $\boldsymbol{x} = (\boldsymbol{x}_1, \boldsymbol{x}_2, \dots, \boldsymbol{x}_p) \in G_\alpha$ exist, such that$\left(f_{1j}(\boldsymbol{x}^*)\right) \leq \left(f_{1j}(\boldsymbol{x})\right)$ with at least one strict inequality hold for$j$ $(j = 1,2,\dots,k_1)$; so $(\boldsymbol{x}_1^*, \boldsymbol{x}_2^*, \dots, \boldsymbol{x}_p^*)$ is the α-Pareto optimal solution of the ML-MOQFP problem.

4 Nonlinear Model Development of the ML-MOQFP Problem

Now, we make further extensions on the article of Lachhwani [17], to develop a methodology for obtaining the equivalent non-linear model of the ML-MOQFP problem. Since the MOQFP problem for the i^{th}-level decision maker may be written as:

$$\underbrace{max}_{x_i} F_i(\boldsymbol{x}) = \underbrace{max}_{x_i} \left(f_{i1}(\boldsymbol{x}), f_{i2}(\boldsymbol{x}), \dots, f_{ik_i}(\boldsymbol{x})\right), \tag{19}$$

$subject\ to$

$$\boldsymbol{x} \in \boldsymbol{G}_\alpha = \left\{ \boldsymbol{x} \in R^n \middle| \begin{array}{ll} \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_\alpha^U x_j \le \left(\tilde{b}_i\right)_\alpha^U, & x_j \ge 0 \\ \sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_\alpha^L x_j \le \left(\tilde{b}_i\right)_\alpha^L, & i = 1,2,\dots,m \end{array} \right\} \tag{20}$$

Where

$$f_{ij}(\boldsymbol{x}) = \frac{Q_1^{ij}\boldsymbol{x_1^2} + Q_2^{ij}\boldsymbol{x_2^2} + \dots + Q_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{c_1^{ij}x_1} + \boldsymbol{c_2^{ij}x_2} + \dots + \boldsymbol{c_p^{ij}x_p} + \alpha^{ij}}{R_1^{ij}\boldsymbol{x_1^2} + R_2^{ij}\boldsymbol{x_2^2} + \dots + R_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{d_1^{ij}x_1} + \boldsymbol{d_2^{ij}x_2} + \dots + \boldsymbol{d_p^{ij}x_p} + \beta^{ij}} \qquad i = 1,2,\dots p, \quad j = 1,2,\dots,k_i \tag{21}$$

Thus, in contrast to eq. (15) for the sake of simplicity in this paper we employ the representation of eq. (21) in order to deal with the ML-MOQFP problem. Let us take the transformation:

$$y^{ij} = \frac{1}{R_1^{ij}\boldsymbol{x_1^2} + R_2^{ij}\boldsymbol{x_2^2} + \dots + R_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{d_1^{ij}x_1} + \boldsymbol{d_2^{ij}x_2} + \dots + \boldsymbol{d_p^{ij}x_p} + \beta^{ij}}, \tag{22}$$

which is equivalent to:

$$y^{ij}\left(R_1^{ij}\boldsymbol{x_1^2} + R_2^{ij}\boldsymbol{x_2^2} + \dots + R_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{d_1^{ij}x_1} + \boldsymbol{d_2^{ij}x_2} + \dots + \boldsymbol{d_p^{ij}x_p} + \beta^{ij}\right) = 1, \tag{23}$$

So, each quadratic fractional objective function is transformed into the following equivalent non-linear function

$$f_{ij}(\boldsymbol{x}, y) = \left(Q_1^{ij}\boldsymbol{x_1^2} + Q_2^{ij}\boldsymbol{x_2^2} + \dots + Q_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{c_1^{ij}x_1} + \boldsymbol{c_2^{ij}x_2} + \dots + \boldsymbol{c_p^{ij}x_p} + \alpha^{ij}\right)y^{ij} \quad \forall\, i,j \tag{24}$$

Based on the equation (24), the non-linear model of the MOQFP problem for i^{th} level decision maker is formulated as follows:

$$\underbrace{max}_{x_i} f_{ij}(\boldsymbol{x}, y) = \left[Q_1^{ij}\boldsymbol{x_1^2} + Q_2^{ij}\boldsymbol{x_2^2} + \dots + Q_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{c_1^{ij}x_1} + \boldsymbol{c_2^{ij}x_2} + \dots + \boldsymbol{c_p^{ij}x_p} + \alpha^{ij}\right]y^{ij}, \tag{25}$$

$subject\ to$

$$y^{ij}\left[R_1^{ij}\boldsymbol{x_1^2} + R_2^{ij}\boldsymbol{x_2^2} + \dots + R_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{d_1^{ij}x_1} + \boldsymbol{d_2^{ij}x_2} + \dots + \boldsymbol{d_p^{ij}x_p} + \beta^{ij}\right] = 1, \qquad \forall i,j \tag{26}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_\alpha^U x_j \le \left(\tilde{b}_i\right)_\alpha^U, \qquad x_j \ge 0 \tag{27}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_{\alpha}^{L} x_j \leq \left(\tilde{b}_i\right)_{\alpha}^{L}, \qquad i = 1,2, \dots, m \tag{28}$$

Following the above discussion thus, the ML-MONP model of the ML-MOQFP problem is formulated as follows:

$[\boldsymbol{1^{st}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_1}} F_1(\boldsymbol{x}, y) = \underbrace{max}_{\boldsymbol{x_1}} \left(f_{11}(\boldsymbol{x}, y), f_{12}(\boldsymbol{x}, y), \dots, f_{1k_1}(\boldsymbol{x}, y)\right), \tag{29}$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3}, \dots, \boldsymbol{x_p}\ \ solves$

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_2}} F_2(\boldsymbol{x}, y) = \underbrace{max}_{\boldsymbol{x_2}} \left(f_{21}(\boldsymbol{x}, y), f_{22}(\boldsymbol{x}, y), \dots, f_{2k_2}(\boldsymbol{x}, y)\right), \tag{30}$$

$\vdots$

$where\ \boldsymbol{x_p}\ \ solves$

$[\boldsymbol{p^{th}\ Level}]$

$$\underbrace{max}_{\boldsymbol{x_p}} F_p(\boldsymbol{x}, y) = \underbrace{max}_{\boldsymbol{x_p}} \left(f_{p1}(\boldsymbol{x}, y), f_{p2}(\boldsymbol{x}, y), \dots, f_{pk_p}(\boldsymbol{x}, y)\right), \tag{31}$$

$subject\ to$

$$y^{ij}\left[R_1^{ij}\boldsymbol{x_1^2} + R_2^{ij}\boldsymbol{x_2^2} + \dots + R_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{d_1^{ij}x_1} + \boldsymbol{d_2^{ij}x_2} + \dots + \boldsymbol{d_p^{ij}x_p} + \beta^{ij}\right] = 1, \qquad \forall i, j \tag{32}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_{\alpha}^{U} x_j \leq \left(\tilde{b}_i\right)_{\alpha}^{U}, \qquad x_j \geq 0 \tag{33}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_{\alpha}^{L} x_j \leq \left(\tilde{b}_i\right)_{\alpha}^{L}, \qquad i = 1,2, \dots, m \tag{34}$$

Where

$$f_{ij}(\boldsymbol{x}, y) = \left[Q_1^{ij}\boldsymbol{x_1^2} + Q_2^{ij}\boldsymbol{x_2^2} + \dots + Q_p^{ij}\boldsymbol{x_p^2} + \boldsymbol{c_1^{ij}x_1} + \boldsymbol{c_2^{ij}x_2} + \dots + \boldsymbol{c_p^{ij}x_p} + \alpha^{ij}\right]y^{ij}, \quad \forall i, j$$

and the system of constraints, in equations (32)-(34), at an α-level denoted by $\boldsymbol{S}_{\alpha}$, which form a nonempty convex set.

5 Fuzzy Goal Programming Approach for ML-MOQFP Problem

In the proposed FGP approach, in order to obtain the compromise solution which is a Pareto optimal solution. The vector of non-linear objective functions of the model (29)-(34) for each DM is formulated as a fuzzy goal characterized by its membership function.

5.1 Characterization of membership functions

To define the membership functions of the fuzzy goals [2,36], each objective function's individual maximum is taken as the corresponding aspiration level, as follows:

$$u_{ij} = \underbrace{max}_{x\in S_\alpha} f_{ij}(\boldsymbol{x}, y), \quad (i = 1,2, \dots, p), (j = 1,2, \dots, k_i). \tag{35}$$

where u_{ij}, $(i = 1,2, \dots, p)$, $(j = 1,2, \dots, k_i)$, give the upper tolerance limit or aspired level of achievement for the membership function of ij^{th} objective function. Similarly, each objective function's individual minimum is taken as the corresponding aspiration level, as follows:

$$g_{ij} = \underbrace{min}_{x\in s_\alpha} f_{ij}(\boldsymbol{x}, y), \quad (i = 1,2, \dots, p), (j = 1,2, \dots, k_i). \tag{36}$$

where g_{ij}, $(i = 1,2, \dots, p)$, $(j = 1,2, \dots, k_i)$, give the lower tolerance limit or lowest acceptable level of achievement for the membership function of ij^{th} objective function. It can be assumed reasonably that the values of $f_{ij}(\boldsymbol{x}, y) \geq u_{ij}$, $(i = 1,2, \dots, p)$, $(j = 1,2, \dots, k_i)$, are acceptable and all values less than $g_{ij} = \underbrace{min}_{x\in S_\alpha} f_{ij}(\boldsymbol{x}, y)$, are absolutely unacceptable. Then, the membership function $\mu_{ij}\left(f_{ij}(\boldsymbol{x}, y)\right)$, for the ij^{th} fuzzy goal can be formulated as:

$$\mu_{ij}\big(f \ \ (\boldsymbol{x}, y)\big) = \begin{cases} 1, & if \ f_{ij}(\boldsymbol{x}, y) \geq u_{ij}, \\ \dfrac{f_{ij}(\boldsymbol{x}, y) - g_{ij}}{u_{ij} - g_{ij}}, & if \ \ g_{ij} \leq f_{ij}(\boldsymbol{x}, y) \leq u_{ij}, (i = 1, \dots, p), (j = 1, \dots, k_i), \\ 0, & if \ f_{ij}(\boldsymbol{x}, y) \leq g_{ij}, \end{cases} \tag{37}$$

5.2 Fuzzy goal programming model

In the decision-making context, each decision maker is interested in maximizing his or her own objective function; the optimal solution of each DM, when calculated in isolation, would be considered as the best solution and the associated value of the objective function can be considered as the aspiration level of the corresponding fuzzy goal. In fuzzy programming approach, the highest degree of membership is one [22]. So, for the defined membership functions in equations (37), the flexible membership goals having the aspired level unity can be represented as follows:

$$\mu_{f_{ij}}\left(f_{ij}(\boldsymbol{x}, y)\right) + \ d_{ij}^- - d_{ij}^+ = 1, \quad (i = 1,2, \dots, p), \ \ (j = 1,2, \dots, k_i), \tag{38}$$

or equivalently as:

$$\frac{f_{ij}(\boldsymbol{x}, y) - g_{ij}}{u_{ij} - g_{ij}} + d_{ij}^- - d_{ij}^+ = 1, \qquad (i = 1,2, \dots, p), \ \ (j = 1,2, \dots, k_i), \tag{39}$$

where $d_{ij}^-, d_{ij}^+ \geq 0$, with $d_{ij}^- \times d_{ij}^+ = 0$, represent the under- and over- deviations, respectively, from the aspired levels [20].

Following the basic concept of MLMP, the FLDM decides his/her objectives and/or choices, hence asks each inferior level of the association for their solutions, which obtained individually. The lower level decision makers' choices are then presented and altered by the FLDM in light of the general advantage for the organization. Thus, the vector of decision variables $\boldsymbol{x}_{il}$, $(i = 1,2, \dots, p - 1)$, $(l = 1,2, \dots, n_i)$, for the top levels are taken as a binding constraintsfor the p^{th}-level problem as follows:

$$\boldsymbol{x}_{il} = \boldsymbol{x}_{il}^{*}(i = 1,2, \dots, p-1), \quad (l = 1,2, \dots, n_i). \tag{40}$$

In the classical methodology of goal programming, the under- and over- deviational variables are included in the achievement function for minimizing them depends upon the type of the objective functions to be optimized. In the proposed FGP approach, the sum of under deviational variables is required to be minimized to achieve the aspired level. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value [2,36]. Thus considering the goal achievement problem at the same priority level, the proposed final FGP model for the ML-MONP problem follows as:

$$min \qquad Z = \sum_{j=1}^{k_1} w_{1j}^{-} d_{1j}^{-} + \sum_{j=1}^{k_2} w_{2j}^{-} d_{2j}^{-} + \dots + \sum_{j=1}^{k_p} w_{pj}^{-} d_{pj}^{-}, \tag{41}$$

$subject\ to$

$$\frac{f_{ij}(\boldsymbol{x}, y) - g_{ij}}{(u_{ij} - g_{ij})} + d_{ij}^{-} - d_{ij}^{+} = 1, \qquad i = 1,2, \dots, p, \quad j = 1,2, \dots, k_i, \tag{42}$$

$$y^{ij}\left[R_1^{ij}\boldsymbol{x}_1^2 + R_2^{ij}\boldsymbol{x}_2^2 + \dots + R_p^{ij}\boldsymbol{x}_p^2 + \boldsymbol{d}_1^{ij}\boldsymbol{x}_1 + \boldsymbol{d}_2^{ij}\boldsymbol{x}_2 + \dots + \boldsymbol{d}_p^{ij}\boldsymbol{x}_p + \beta^{ij}\right] = 1, \qquad \forall i, j \tag{43}$$

$$\boldsymbol{x}_{il} = \boldsymbol{x}_{il}^{*}(i = 1,2, \dots, p-1), \quad (l = 1,2, \dots, n_i). \tag{44}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_{\alpha}^{U} x_j \le \left(\tilde{b}_i\right)_{\alpha}^{U}, \qquad x_j \ge 0 \tag{45}$$

$$\sum_{j=1}^{n} \left(\tilde{A}_{ij}\right)_{\alpha}^{L} x_j \le \left(\tilde{b}_i\right)_{\alpha}^{L}, \qquad i = 1,2, \dots, m \tag{46}$$

$$d_{ij}^{-} \times d_{ij}^{+} = 0, and\ d_{ij}^{-}, d_{ij}^{+} \ge 0, \qquad (i = 1,2, \dots, p), \ (j = 1,2, \dots, k_i), \tag{47}$$

where$Z$ represents the achievement function consisting of the weighted under-deviational variables of the fuzzy goals. The numerical weightsw_{ij}^{-}represent the relative importance of achieving the aspired levels of the respective fuzzy goals. To assess the relative importance of the fuzzy goals properly, the weighting scheme suggested in [12] is used to assign the values to,w_{ij}^{-}. These values are determined as:

$$w_{ij}^{-} = \frac{1}{u_{ij} - g_{ij}}, \qquad (i = 1,2, \dots, p), (j = 1,2, \dots, k_i), \tag{48}$$

6 The FGP Algorithm for ML-MOQFP Problem with Fuzzy parameters

Following the above discussion, the proposed FGP algorithm will be constructed for solving the ML-MOQFP problems with fuzzy parameters as follows:

Step 1. Set the value ofα,acceptable for all decision makers, for the degree of all membership functions of the fuzzy parameters.

Step 2. Formulate the crisp set of constraints for the ML-MOQFP problem at the givenα-level, equation (18).

Step 3. Formulate the ML-MONP model, equation (29)-(34), of the ML-MOQFP problem.

Step 4. Calculate the individual maximum and minimum values for each objective function $f_{ij}(x, y)$ in all levels subject to the set of constraints.

Step 5. Set the goals and the upper tolerance limits for each objective function in all levels.

Step 6. Evaluate the weights w_{ij} as defined in equation (48).

Step 7. Set $l = 1$, for the i^{th} level decision-making problem.

Step 8. Build the membership functions $\mu_{ij}(f_{ij}(x, y))$ $\;j = 1,2, \dots, m\_l$, as in equation (37).

Step 9. Solve the i^{th} -level FGP model sequentially to get $\boldsymbol{x_{il}} = \boldsymbol{x_{il}^*}$.

Step 10. If $l > t - 1$, then go to the Step 11; otherwise set $l = l + 1$, and go to Step 8.

Step 11. Solve the final FGP model for the ML-MOQFP problem with fuzzy parameters.

Step 12. If all decision makers are satisfied with the compromise solution in Step11; then go to Step 14;otherwise go to Step 13.

Step13. Improve the upper and lower tolerance limitsu_{ij}, g_{ij},for all objective goals in all levels, go to Step 6.

Step14. Stop with the satisfactory solution for all decision makers in the problem.

7 Illustrative Example

To demonstrate the proposed FGP approach, consider the following ML-MOQFP problem with fuzzy parameters in the constraints.

$[\boldsymbol{1^{st}\ Level}]$

$$\underbrace{max}_{x_1}\left(f_{11} = \frac{-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10}{x_1^2 + 3x_2 + 5}, \quad f_{12} = \frac{-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12}{x_1^2 + 3x_2 + 5}\right),$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3} solves$

$[\boldsymbol{2^{nd}\ Level}]$

$$\underbrace{max}_{x_2}\left(f_{21} = \frac{-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6}{x_2^2 + 3x_1 + 1}, \quad f_{22} = \frac{-3x_1^2 - x_2^2 - x_3^2 + 7x_3 + 8}{x_2^2 + 3x_1 + 1}\right)$$

$where\ \boldsymbol{x_3} solves$

$[\boldsymbol{3^{rd}\ Level}]$

$$\underbrace{max}_{x_3}\left(f_{31} = \frac{-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7}{x_3^2 + 5x_2 + 2}, \quad f_{32} = \frac{-x_1^2 - 2x_2^2 - x_3^2 + 9}{x_3^2 + 5x_2 + 2}\right),$$

$subject\ to$

$\tilde{4}x_1 + \tilde{7}x_2 + \tilde{2}x_3 \leq \widetilde{30},$

$\tilde{3}x_1 - \tilde{0}x_2 + \widetilde{14}x_3 \leq \widetilde{18},$

$\tilde{7}x_2 + \tilde{8}x_3 \geq \widetilde{12},$

Here, the fuzzy numbers are assumed to be $\mathcal{LR}$-fuzzy numbers and are given as follows:

$\tilde{4} = (4,2,1)_{LR}$,$\tilde{7} = (7,4,2)_{LR}$,$\tilde{2} = (2,2,3)_{LR}$,$\tilde{3} = (3,2,2)_{LR}$,$\tilde{0} = (0,1,2)_{LR}$,$\widetilde{14} = (14,4,2)_{LR}$, $\tilde{8} = (8,4,2)_{LR}$, $\widetilde{30} = (30,5,10)_{LR}$, $\widetilde{18} = (18,3,4)_{LR}$, $\widetilde{12} = (12,2,8)_{LR}$

Following the proposed FGP approach, the solution of the ML-MOQFP problem with fuzzy parameters obtained at a desired value of α, assume that an α-level of 0.8 is accepted by the three level DMs.Thus the deterministic model of the ML-MOQFP problemisobtained as follows:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\left(f_{11} = \frac{-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10}{x_1^2 + 3x_2 + 5},\quad f_{12} = \frac{-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12}{x_1^2 + 3x_2 + 5}\right),$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3} solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\left(f_{21} = \frac{-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6}{x_2^2 + 3x_1 + 1},\quad f_{22} = \frac{-3x_1^2 - x_2^2 - x_3^2 + 7x_3 + 8}{x_2^2 + 3x_1 + 1}\right)$$

$where\ \boldsymbol{x_3} solves$

$[\mathbf{3^{rd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_3}\left(f_{31} = \frac{-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7}{x_3^2 + 5x_2 + 2},\quad f_{32} = \frac{-x_1^2 - 2x_2^2 - x_3^2 + 9}{x_3^2 + 5x_2 + 2}\right),$$

$subject\ to$

$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$

$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$

$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$

$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$

$6.2x_2 + 7.2x_3 \geq 11.6,$

$7.4x_2 + 8.4x_3 \geq 13.6,$

Then the ML-MOQFP problem is transformed into ML-MONP model based on the proposed transformation as follows:

$[\mathbf{1^{st}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_1}\begin{pmatrix} f_{11}(x,y) = (-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10)y_1, \\ f_{12}(x,y) = (-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12)y_1, \end{pmatrix},$$

$where\ \boldsymbol{x_2}, \boldsymbol{x_3} solves$

$[\mathbf{2^{nd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_2}\begin{pmatrix} f_{21}(x,y) = (-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6)y_2, \\ f_{22}(x,y) = (-3x_1^2 - x_2^2 - x_3^2 + 7x_2 + 8)y_2, \end{pmatrix}$$

$where\ \boldsymbol{x_3} solves$

$[\mathbf{3^{rd}}\ \boldsymbol{Level}]$

$$\underbrace{max}_{x_3}\begin{pmatrix} f_{31}(x,y) = (-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7)y_3, \\ f_{32}(x,y) = (-x_1^2 - 2x_2^2 - x_3^2 + 9)y_3, \end{pmatrix},$$

$subject\ to$

$$(x_1^2 + 3x_2 + 5)y_1 = 1,$$

$$(x_2^2 + 3x_1 + 1)y_2 = 1,$$

$$(x_3^2 + 5x_2 + 2)y_3 = 1,$$

$$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$$

$$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$$

$$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$$

$$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$$

$$6.2x_2 + 7.2x_3 \geq 11.6,$$

$$7.4x_2 + 8.4x_3 \geq 13.6,$$

Table 1. Individual maximum, minimum values, u_{ij}, g_{ij} and weights w_{ij}

	$\boldsymbol{f_{11}(x,y)}$	$\boldsymbol{f_{12}(x,y)}$	$\boldsymbol{f_{21}(x,y)}$	$\boldsymbol{f_{22}(x,y)}$	$\boldsymbol{f_{31}(x,y)}$	$\boldsymbol{f_{32}(x,y)}$
$max f_{ij}(\boldsymbol{x},\boldsymbol{y})$	1.632	1.426	7.83	7.88	1.29	1.281
$min f_{ij}(\boldsymbol{x},\boldsymbol{y})$	-0.321	-0.562	-1.59	-2.67	-1.77	-1.2
u_{ij}	1.632	1.426	7.83	7.88	1.29	1.281
g_{ij}	-0.321	-0.562	-1.59	-2.67	-1.77	-1.2
w_{ij}	0.512	0.503	0.106	0.095	0.327	0.403

Therefore, Solve the FGP models sequentially to get $\boldsymbol{x_1} = \boldsymbol{x_1^*}and\boldsymbol{x_2} = \boldsymbol{x_2^*}$. Thus the first level FGP model follows as:

$$minZ = 0.512d_{11}^- + 0.503d_{12}^-,$$

subject to

$$(-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10)y_1 + 1.9533d_{11}^- - 1.953d_{11}^+ = 1.632,$$

$$(-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12)y_1 + 1.988d_{12}^- - 1.988d_{12}^+ = 1.426,$$

$$(x_1^2 + 3x_2 + 5)y_1 = 1,$$

$$(x_2^2 + 3x_1 + 1)y_2 = 1,$$

$$(x_3^2 + 5x_2 + 2)y_3 = 1,$$

$$3.6x_1 + 6.2x_2 + 1.6x_3 \le 29,$$

$$4.2x_1 + 7.4x_2 + 2.6x_3 \le 32,$$

$$2.6x_1 - 0.4x_2 + 13.2x_3 \le 17.4,$$

$$3.4x_1 + 0.2x_2 + 14.4x_3 \le 18.8,$$

$$6.2x_2 + 7.2x_3 \ge 11.6,$$

$$7.4x_2 + 8.4x_3 \ge 13.6,$$

$$d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+ \ge 0,$$

Using Lingo programming, the compromise solution of the first level decision making problem is obtained as; $(x_1, x_2, x_3) = (0.5055, 0.7659, 0.9515)$. Then assuming that the FLDM set$x_1^* = 0.5055$.

The second level decision maker FGP model follows as:

$$minZ = 0.512d_{11}^- + 0.503d_{12}^- + 0.106d_{21}^- + 0.095d_{22}^-,$$

subject to

$$(-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10)y_1 + 1.9533d_{11}^- - 1.953d_{11}^+ = 1.632,$$

$$(-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12)y_1 + 1.988d_{12}^- - 1.988d_{12}^+ = 1.426,$$

$$(-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6)y_2 + 9.42d_{21}^- - 9.42d_{21}^+ = 7.83,$$

$$(-3x_1^2 - x_2^2 - x_3^2 + 7x_2 + 8)y_2 + 10.55d_{21}^- - 10.55d_{21}^+ = 7.88,$$

$$(x_1^2 + 3x_2 + 5)y_1 = 1,$$

$$(x_2^2 + 3x_1 + 1)y_2 = 1,$$

$$(x_3^2 + 5x_2 + 2)y_3 = 1,$$

$$3.6x_1 + 6.2x_2 + 1.6x_3 \le 29,$$

$$4.2x_1 + 7.4x_2 + 2.6x_3 \le 32,$$

$$2.6x_1 - 0.4x_2 + 13.2x_3 \le 17.4,$$

$$3.4x_1 + 0.2x_2 + 14.4x_3 \le 18.8,$$

$$6.2x_2 + 7.2x_3 \ge 11.6,$$

$$7.4x_2 + 8.4x_3 \ge 13.6,$$

$$x_1^* = 0.5055,$$

$$d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+, d_{21}^-, d_{21}^+, d_{22}^-, d_{22}^+ \ge 0,$$

Using Lingo programming, the compromise solution of the second level decision making problem is obtained as; $(x_1, x_2, x_3) = (0.5055,0.7207, 0.99)$. Also, the SLDM sets $x_2^* = 0.7207$.

Hence, the final FGP model for the ML-MOQFP problem with fuzziness in the constraints is obtained as follows:

$$minZ = 0.512d_{11}^- + 0.503d_{12}^- + 0.106d_{21}^- + 0.095d_{22}^- + 0.327d_{31}^- + 0.403d_{32}^-,$$

$subject\ to$

$$(-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10)y_1 + 1.9533d_{11}^- - 1.953d_{11}^+ = 1.632,$$

$$(-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12)y_1 + 1.988d_{12}^- - 1.988d_{12}^+ = 1.426,$$

$$(-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6)y_2 + \ 9.42d_{21}^- - 9.42d_{21}^+ = 7.83,$$

$$(-3x_1^2 - x_2^2 - x_3^2 + 7x_2 + 8)y_2 + 10.55d_{21}^- - 10.55d_{21}^+ = 7.88,$$

$$(-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7)y_3 + 3.06d_{31}^- - 3.06d_{31}^+ = 1.29,$$

$$(-x_1^2 - 2x_2^2 - x_3^2 + 9)y_3 + 2.481d_{32}^- - 2.481d_{32}^+ = 1.281,$$

$$(x_1^2 + 3x_2 + 5)y_1 = 1,$$

$$(x_2^2 + 3x_1 + 1)y_2 = 1,$$

$$(x_3^2 + 5x_2 + 2)y_3 = 1,$$

$$3.\,6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$$

$$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$$

$$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$$

$$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$$

$$6.2x_2 + 7.2x_3 \geq 11.6,$$

$$7.4x_2 + 8.4x_3 \geq 13.6,$$

$$x_1^* = 0.5055,$$

$$x_2^* = 0.7207,$$

$$d_{11}^-, d_{11}^+, d_{12}^-, d_{12}^+, d_{21}^-, d_{21}^+, d_{22}^-, d_{22}^+, d_{31}^-, d_{31}^+, d_{32}^-, d_{32}^+ \geq 0,$$

Using Lingo programming software version 16, the compromise solution of the ML-MOQFP problem is obtained as$(x_1, x_2, x_3) = (0.5055,0.7207, 0.99)$with the corresponding objective function values $f_{11}(\boldsymbol{x}) = 1.527$, $f_{12}(\boldsymbol{x}) = 1.325$, $f_{21}(\boldsymbol{x}) = 2.54$, $f_{22}(\boldsymbol{x}) = 3.55$, $f_{31}(\boldsymbol{x}) = 1.22, f_{32}(\boldsymbol{x}) = 1.02$ .and their corresponding membership function $\mu_{11} = 0.95, \mu_{12} = 0.95, \mu_{21} = 0.438, \mu_{22} = 0.589$, $\mu_{31} = 0.977, \mu_{32} = 0.895$.

Comparison with the existing method; the modified FGP approach presented by K. Lachhwani [6] for solving ML-MOQFP problem follows as:

$min\lambda = d_{11}^{N-} + d_{12}^{N-} + d_{21}^{N-} + d_{22}^{N-} + d_{31}^{N-} + d_{32}^{N-} + d_{11}^{D-} + d_{12}^{D-} + d_{21}^{D-} + d_{22}^{D-} + d_{31}^{D-} + d_{32}^{D-} + d_1^- + d_2^-,$
$subject\ to$

$-x_1^2 - 2x_2^2 - x_3^2 + 5x_2 + 10 + 18.69d_{11}^{N-} \geq 12.91,$

$-x_1^2 - x_2^2 - 4x_3^2 + 5x_1 + 12 + 24.99d_{12}^{N-} \geq 15.63,$

$-x_1^2 - 2x_2^2 - 2x_3^2 + 5x_3 + 6 + 40.258d_{21}^{N-} \geq 8.858,$

$-3x_1^2 - x_2^2 - x_3^2 + 7x_2 + 8 + 67.03d_{22}^{N-} \geq 20.25,$

$-x_1^2 - 4x_2^2 - x_3^2 + 6x_2 + 7 + 50.31d_{31}^{N-} \geq 8.46,$

$-x_1^2 - 2x_2^2 - x_3^2 + 9 + 35.5d_{32}^{N-} \geq 7.1,$

$-x_1^2 - 3x_2 - 5 + 25.01d_{11}^{D-} \geq -6.1,$

$-x_1^2 - 3x_2 - 5 + 25.01d_{12}^{D-} \geq -6.1,$

$-x_2^2 - 3x_1 - 1 + 18.57d_{21}^{D-} \geq -1.13,$

$-x_2^2 - 3x_1 - 1 + 18.57d_{22}^{D-} \geq -1.13,$

$-x_3^2 - 5x_2 - 2 + 18.12d_{31}^{D-} \geq -5.5,$

$-x_3^2 - 5x_2 - 2 + 18.12d_{32}^{D-} \geq -5.5,$

$x_1 + 2.5d_1^- \geq 2.5,$

$x_2 + 1.86d_2^- \geq 3.5,$

$3.6x_1 + 6.2x_2 + 1.6x_3 \leq 29,$

$4.2x_1 + 7.4x_2 + 2.6x_3 \leq 32,$

$2.6x_1 - 0.4x_2 + 13.2x_3 \leq 17.4,$

$3.4x_1 + 0.2x_2 + 14.4x_3 \leq 18.8,$

$6.2x_2 + 7.2x_3 \geq 11.6,$

$7.4x_2 + 8.4x_3 \geq 13.6,$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0,$

$d_{11}^{N-}, d_{12}^{N-}, d_{21}^{N-}, d_{22}^{N-}, d_{31}^{N-}, d_{32}^{N-}, d_{11}^{D-}, d_{12}^{D-}, d_{21}^{D-}, d_{22}^{D-}, d_{31}^{D-}, d_{32}^{D-}, d_1^-, d_2^- \geq 0.$

Using Lingo programming software version 16, the compromise solution of theML-MOQFP problem with fuzzy parameters using the modified FGP [6] is obtained as; $(x_1, x_2, x_3) = (0.476, 0.861, 0.869)$ with objective function values $f_{11}(\boldsymbol{x}) = 1.516, f_{12}(\boldsymbol{x}) = 1.33$, $f_{21}(\boldsymbol{x}) = 2.248$, $f_{22}(\boldsymbol{x}) = 3.739$, $f_{31}(\boldsymbol{x}) = 1.164$, $f_{32}(\boldsymbol{x}) = 0.926$, and their corresponding membership function $\mu_{11} = 0.94, \mu_{12} = 0.95, \mu_{21} = 0.41, \mu_{22} = 0.61$, $\mu_{31} = 0.959, \mu_{32} = 0.857$.

The comparison between the proposed FGP approach, interactive approach and the method presented by Lachhwani [6] is given in Table 2. The results show that the compromise solution of the proposed FGP model and the interactive approach, are preferred than the latter method presented by Lachhwani in [6]. As the proposed FGP model avoids the main shortcomings presented in [6]. Indeed, Lachhwani claimed that the ratio optimization (fractional programming) problems are equivalent to a multi-objective programming problem. As he considered the objective numerators as maximization problems; so also are the objective denominators as minimization problems.

Table 2. Comparison between the FGP approach and Lachhwani [6]

Proposed FGP approach		Interactive approach (underreview)		Lachhwani [6]	
$f_{11} = 1.527$	$\mu_{11} = 0.95$	$f_{11} = 1.593$	$\mu_{11} = 0.98$	$f_{11} = 1.516$	$\mu_{11} = 0.94$
$f_{12} = 1.325$	$\mu_{12} = 0.95$	$f_{12} = 1.113$	$\mu_{12} = 0.843$	$f_{12} = 1.33$	$\mu_{12} = 0.95$
$f_{21} = 2.54$	$\mu_{21} = 0.438$	$f_{21} = 4.154$	$\mu_{21} = 0.61$	$f_{21} = 2.248$	$\mu_{21} = 0.41$
$f_{22} = 3.55$	$\mu_{22} = 0.589$	$f_{22} = 7.034$	$\mu_{22} = 0.92$	$f_{22} = 3.739$	$\mu_{22} = 0.61$
$f_{31} = 1.22$	$\mu_{31} = 0.977$	$f_{31} = 1.27$	$\mu_{31} = 0.99$	$f_{31} = 1.164$	$\mu_{31} = 0.959$
$f_{32} = 1.02$	$\mu_{32} = 0.895$	$f_{32} = 1.223$	$\mu_{32} = 0.97$	$f_{32} = 0.926$	$\mu_{32} = 0.857$

8 Summary and Conclusion

This paper reveals how the concept of FGP approach can be efficiently used for solving ML-MOQFP problems with fuzzy parameters. Based on theα-level properties and partial order relation, a numerical general model is constructed. An effort has been made to solve the ML-MOQFP problem with fuzzy parameters based on the fuzzy set theory and goal programming approach. Thus, the numerical results for the given example obtained to validity of the proposed method. The FGP approach appears to be promising and computationally easy to implement.

However there are many open points for discussion in future, which should be explored in the area of multi-level quadratic fractional optimization such as:

1. Interactive algorithm is needed for dealing with multi-level multi-objective quadratic fractional programming with fuzzy parameters.
2. Fuzzy goal programming algorithm is required for treating multi-level integer multi-objective quadratic fractional with fuzzy parameters.
3. Fuzzy goal programming algorithm is required for treating multi-level integer multi-objective quadratic fractional in rough environment.

Competing Interests

Authors have declared that no competing interests exist.

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